Probing clouds in planets with a simple Radiative Transfer model: the Jupiter case

Iñigo Mendikoa Alonso, Santiago Pérez-Hoyos and Agustín Sánchez-Lavega

Dpto. Física Aplicada I, E.T.S. Ingeniería, Universidad del País Vasco UPV/EHU (Spain)

E-mail: imendikoa@gmail.com

Abstract. Remote-sensing of planets evokes using expensive on-orbit satellites and gathering complex data from space. However, the basic properties of clouds in planetary atmospheres can be successfully estimated with small telescopes even from an urban environment using currently available and affordable technology. This makes the process accessible for under-graduate students while preserving most of the physics and mathematics involved. This paper presents the methodology for carrying out a photometric study of planetary atmospheres, focused on planet Jupiter. The method introduces the basics of radiative transfer in planetary atmospheres, some notions on inverse problem theory and fundamentals of planetary photometry. As it will be shown, the procedure allows the student to derive the cloud spectral reflectivity and cloud top altitude from observations at different wavelengths by applying a simple but enlightening “reflective layer model”. In this way, the planet atmosphere structure is estimated by students as an inverse problem from the observed photometry. Web resources are also provided to help those unable to get telescopic observations of the planets.

Physics and Astronomy Classification Scheme (PACS) indexing codes. Planets atmosphere: *96.15.H-, 96.15.Hy. Radiative transfer in atmosphere: 42.68.Ay, 92.60.Vb

Submitted to European Journal of Physics, 27 May 2012

1. Introduction

In recent years, small observatories have gained great importance in professional training in Astronomy and Space Sciences, as well as in certain research activities that the large number and wide availability of those observatories allow. The key to this development is rooted in the CCD technology lower prices and in the increased capacities of both detectors and processors. Today, completely robotized professional and amateur observatories are a reality and actively collaborate in areas as detection of transits of extrasolar planets (e.g. Pollaco et al., 2006) or the study of planets in our solar system (e.g., Sánchez-Lavega et al., 1991, 2008, 2010, 2011; Hueso et al., 2010a). To this particular respect, the collaboration between amateur and professional astronomers has proved to be an excellent way to acquire the desired temporal resolution for many planetary transient phenomena (e.g., see Hueso et al., 2010b).

The activity here described has a direct application as a practice lesson for under-graduate students. We propose to estimate main characteristics of a planetary atmosphere as an inverse problem from the photometry that can be measured from Earth using a small telescope. This methodology is validated with the results presented for the case of Jupiter, providing pressure level values of the upper reflective clouds as well as their reflectivity at the different wavelengths studied.

The following sections describe the different steps of the whole process, starting from the description of the theoretical radiative model used for the above mentioned inverse problem solving. The equipment main characteristics are described afterwards, including the necessary parameters to be used in the photometric analysis of Jupiter atmosphere and, finally, some results obtained for Jupiter’s atmospheric structure are presented, which are acceptable according to relevant data available in the literature.
2. Radiative transfer model for planetary atmospheres

2.1. Basic equations

When radiation of wavelength $\lambda$ passes through a medium of thickness $ds$, its intensity is attenuated by both scattering and absorption processes according to this expression (e.g. Sanchez-Lavega, 2011):

$$dI_\lambda = -I_\lambda \rho \kappa_\lambda ds$$  \hspace{1cm} (1)

where $\rho$ is the medium density and $\kappa_\lambda$ the extinction coefficient (area per mass units). If we consider radiation in a planetary atmosphere propagating in a direction given by $\mu$, cosine of the zenith angle $\theta$, the intensity attenuation can be expressed in this way:

$$dI_\lambda = -I_\lambda \frac{d\tau_\lambda}{\mu}$$  \hspace{1cm} (2)

in terms of the optical depth

$$d\tau_\lambda = \rho \kappa_\lambda dz$$  \hspace{1cm} (3)

where $z$ represents the atmospheric depth. This assumes no radiation sources in the atmosphere.

Let’s for now neglect multiple scattering and assume that the additional radiation source term is negligible and we can easily solve the resulting simplified radiative transfer equation. Considering a homogeneous medium this expression leads to the well known Beer-Lambert law representing the exponential decay of the incoming radiation intensity $I_{\lambda0}$ passing through a homogeneous medium:

$$I_\lambda = I_{\lambda0} e^{-\tau_\lambda}$$  \hspace{1cm} (4)

Let us now suppose that there is a reflecting layer located at a certain depth in the planetary atmosphere. We assume that this layer reflects a given fraction $R$ of the incoming normal radiation. For some cases (e.g. Mars) we could assume that the reflecting layer is the planetary surface but in our case (focused on Jupiter), it will represent the uppermost cloud deck. The incident sunlight with intensity $I_{\lambda0}$ is attenuated when passing through the atmosphere at a zenith angle $\theta_0$, then reflected by the upper cloud with wavelength dependant reflectivity $R_\lambda$ and finally attenuated again in its way back to the outer space, as shown in figure 1. So the incident perpendicular intensity on the cloud is $\mu_0 I$ (where $\mu_0$ is the cosine of the zenith angle $\theta_0$ of the incident radiation) and the reflected intensity $I'$, assumed isotropic (i.e. equal in all directions), is then given by:

$$I' = I_{\lambda0} R_\lambda$$  \hspace{1cm} (5)

We omit hereafter the subscript $\lambda$ in the intensity expressions as we now consider the radiation intensity observed through a given bandwidth corresponding to a characteristic $\tau$ and $R_\lambda$, as described later on. According to the Beer-Lambert law, equation (5) can be expressed in this way:

$$I' = I_{\lambda0} \mu_0 R_\lambda e^{-\tau_0}$$  \hspace{1cm} (6)
The reflected intensity is again attenuated in the way back through the gas, leading to the following expression for the intensity finally reflected by the atmosphere (\( \mu \) representing the cosine of the zenith angle \( \theta \) of reflected radiation towards the observer).

\[
I'' = I e^{-\tau/\mu} = I_0 \mu_0 R_A e^{-\tau/\mu_{\mu_0}} = I_0 \mu_0 R_A e^{-\tau/\mu_{\mu_0}}
\]  
(7)

The values for the cosines \( \mu \) and \( \mu_0 \) depend on the observation/illumination geometry. They can be calculated using local latitude and longitude together with other parameters provided by planetary ephemeris, according to the following expressions (Horak, 1950):

\[
\begin{align*}
\mu &= \sin(B)\sin(\varphi_{pg}) + \cos(B)\cos(\varphi_{pg})\cos(\Delta L) \\
\mu_0 &= \sin(B')\sin(\varphi_{pg}) + \cos(B')\cos(\varphi_{pg})\cos(\Delta L - \alpha)
\end{align*}
\]  
(8a) and (8b)

where \( B \) is the sub-observer planetocentric latitude, \( B' \) the sub-solar planetocentric latitude, \( \varphi_{pg} \) the local planetographic latitude of a given location on the planet, \( \Delta L \) its local longitude with respect to the central (sub-observer) meridian according to the given reference system and \( \alpha \) the phase angle given the position of the Sun and observer (Earth) with respect to the planet. For a detailed explanation on planetocentric and planetographic latitudes see Sánchez-Lavega (2011).

Since the solar radiation is isotropic, incident radiation on the planet is usually given by \( \pi F \), so denoting from now on the intensity reflected by the atmosphere as \( I \) we can finally express equation (7) in a more usual fashion for radiative analysis of planetary atmospheres using the reflectivity \( I/F \):

\[
\frac{I}{F} = \mu_0 R_A e^{-\tau/\mu_{\mu_0}}
\]  
(9)

This is the so called 'Reflective Layer Model' RLM (e.g. Chandrasekhar, 1960; Hansen&Travis, 1974) where the upper clouds are assumed to be the reflecting surface with a characteristic reflectivity (dependent on wavelength) and located at a position given by the optical depth (at late instance univocally related to the pressure level or physical depth). By comparing the observed absolute reflectivity \( I/F \) at any point in the planetary disk with that predicted by equation (9), the pressure and reflectivity spectrum can be determined as shown in the following sections, as long as composition of the upper atmosphere is known.

We are assuming here that the same upper cloud is responsible for the 'surface reflection' at all wavelengths analyzed, so that we are not probing different altitude clouds at different
wavelengths and only one pressure level is considered at each planet latitude regardless the wavelength analyzed.

2.2. The case of Jupiter

In the case of Jupiter, the reflective layer can be identified as the upper cloud deck putatively formed by ammonia ice or, more likely, the tropospheric haze above it (West et al., 2004). In the visual wavelengths the main extinction sources for the gas are Rayleigh scattering (mainly due to hydrogen and helium) and absorption due to methane. The total optical depth $\tau$ (dependent on the pressure and wavelength) is therefore calculated as the sum of that of Rayleigh scattering and gas absorption (i.e. $\tau = \tau_R + \tau_A$), following these expressions (e.g. Sanchez-Lavega, 2011):

$$\tau_R = \frac{10^6 P \cdot N_a}{\mu_m g} \left[ n(H_2) \cdot \sigma(H_2) + n(He) \cdot \sigma(He) \right]$$

(10)

$$\tau_A = \frac{22.4 \cdot 10^4 P \cdot n(CH_4) \cdot k}{\mu_m g}$$

(11)

where $P$ is the pressure in bar, $N_a$ the Avogadro number, $k$ the methane absorption coefficient in $1$/km-amagat (dependent on wavelength), $\mu_m$ the mean molecular weight of the atmosphere gas in grams per mol, $n(X)$ the relative abundance of component $X$ of the atmosphere and $\sigma$ the respective cross-section in cm$^2$, given for Hydrogen by (Dalgarno & Williams, 1962):

$$\sigma(H_2) = \frac{8.14 \times 10^{-29}}{\lambda^4} + \frac{1.28 \times 10^{-30}}{\lambda^5} + \frac{1.61 \times 10^{-32}}{\lambda^6}$$

(12)

where $\lambda$ is given in $\mu$m, and for Helium:

$$\sigma(He) = 0.05 \cdot \sigma(H_2)$$

(13)

Figure 2 depicts cross-sections for hydrogen and helium (left) and the methane absorption coefficient spectrum (right) where the maximum absorption wavelengths are clearly visible (Karkoschka, 2010). In the estimation of $k$ and $\sigma$ for each image analysis the specific wavelength interval of the filter used during the observation has to be considered or, ideally, the convolution of the spectrum with the filter response.

Finally, $g$ is the gravitational acceleration which depends on the latitude considered $\varphi_p$, planet radius $R_e$ and mass $M$, planet flattening $a$ and rotation period $T_{rot}$ according to this expression (Sánchez-Lavega, 2011):
\[
g(\varphi_{pc}) = \frac{GM}{R_e^2(1 - a \cdot \text{sen}^2 \varphi_{pc})^2} + \frac{4\pi^2}{T_{rot}^2} R_e \left(1 - a \cdot \text{sen}^2 \varphi_{pc}\right) \cos \varphi_{pc}
\]

(14)

In the case of Jupiter we have: \(M = 1,901 \times 10^{24}\) Kg, \(R_e = 71,541,070\) m, \(a = 0.06492\), \(T_{rot} = 35,730\) s, \(\mu_m = 2.22\) gr/mol, \(n(H2) = 0.839\), \(n(He) = 0.156\), \(n(CH4) = 0.002\) (Sánchez-Lavega, 2011).

3. Data gathering and analysis

The theoretical radiative model described above predicts, following equation (9), the absolute reflectivity \(I/F\) at every single point of the planetary disk at a given latitude and wavelength analyzed, provided values for cloud reflectivity \(R_\lambda\) and pressure \(P\) values. By comparing model predictions \((I/F)_{mod}\) with observed values \((I/F)_{obs}\), we can ultimately get an estimation of cloud pressure \(P\) and reflectivity \(R_\lambda\) (at the wavelength observed with the telescope filters) by finding the best match between models and data. This is usually termed an “inverse problem”. Direct problem is solved when calculating the model predicted values of observed phenomena from a given set of parameters describing the atmosphere, while inverse problem is solved when we find the set of parameters that best reproduces observed phenomena. A complete discussion on inverse problems applied to atmospheric retrievals can be found in Rodgers (2000). Since our theoretical model is simple, we will use a simplified approach to the solution. This analysis will be repeated for every planetary latitude, finally providing the overall planetary upper atmosphere structure.

The overall methodology divided in five steps is presented in figure 3. We will discuss separately data acquisition and photometric calibration (steps 1 to 4) and the strategy for solving the inverse problem (last step). In this way we are separating the experimental approach (which is not unique) from the formal solution of the inverse problem (explained here in more detail).

3.1. Data acquisition

The classical way to get good images of a planet is to use a CCD camera together with a good selection of filters. Typically R, G and B broad color filters can be used, together with narrow CH\(_4\) and U in order to properly analyze absorption and scattering processes in the target atmosphere. The only concern regarding filters is that the narrow ones require greater telescope apertures and/or longer exposures. In any case, the spectral response of each specific filter used has to be precisely determined (this is usually provided by the manufacturer).

Lately, video capture has become more popular than single-exposure photography. The common technique is to record up to a couple of minutes for each filter and then use some software processing to retrieve a high signal to noise image. This technique is usually called ‘Lucky imaging’ (Law et al., 2006). The sequences are processed in order to perform the frames alignment, selection, stacking and optimization of signal to noise ratio. Additional image processing operations, like histogram or gamma correction, should be avoided since the photometry would not be preserved. One popular and free software option used to perform the image processing is REGISTAX\(^1\).

\(^1\) REGISTAX: http://www.astronomie.be/REGISTAX/
Next step is called ‘image navigation’ and provides planetary longitude, latitude and photometry for each image pixel. Two options are proposed to the reader: LAIA\(^2\) software or, alternatively, WinJupos\(^3\). The first one is able to extract directly photometry at a given latitude or longitude (in terms of DNs or ‘digital numbers’, still to be calibrated). The second one produces instead a cylindrical projection which could be read by a general purpose software (Matlab, IDL, Scilab or Octave are common choices) to retrieve the brightness value at each pixel. We provide a simple program for this purpose in the companion website (see appendix).

### Table 1. Filter description

<table>
<thead>
<tr>
<th>Name</th>
<th>(\lambda_{\text{eff}}) (nm)</th>
<th>FWHM (nm)</th>
<th>(k_{\text{CH}_4}) (1/km-am)</th>
<th>(\sigma_{\text{H}_2}) (cm(^2))</th>
<th>(A_{\text{IF}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>357</td>
<td>59</td>
<td>0.0000</td>
<td>5.68 \times 10^{-27}</td>
<td>48.7894</td>
</tr>
<tr>
<td>B</td>
<td>456</td>
<td>121</td>
<td>0.0026</td>
<td>2.04 \times 10^{-27}</td>
<td>55.3359</td>
</tr>
<tr>
<td>G</td>
<td>538</td>
<td>84</td>
<td>0.0180</td>
<td>1.03 \times 10^{-27}</td>
<td>68.1999</td>
</tr>
<tr>
<td>R</td>
<td>635</td>
<td>103</td>
<td>0.0874</td>
<td>5.20 \times 10^{-28}</td>
<td>72.4272</td>
</tr>
<tr>
<td>CH4</td>
<td>892</td>
<td>15</td>
<td>21.8308</td>
<td>1.31 \times 10^{-28}</td>
<td>14.3732</td>
</tr>
</tbody>
</table>

---

\(^2\) LAIA: [http://www.astrogea.org/soft/laia/laia.htm](http://www.astrogea.org/soft/laia/laia.htm)

\(^3\) WinJupos: [http://www.grischa-hahn.homepage.t-online.de/astro/winjupos/index.htm](http://www.grischa-hahn.homepage.t-online.de/astro/winjupos/index.htm)
Navigation always requires some extra information such as date, time and location of the observer. The telescope used to get the results presented in this article is a CDK20\(^4\) (Corrected Dall-Kirkham 20") with 508 mm diameter and 3454 mm focal distance. The observatory\(^5\) is located at ‘Escuela Técnica Superior de Ingeniería’ (UPV-EHU) in Bilbao (Spain). Nevertheless this activity can also be performed with a more affordable telescope of 200 mm diameter or even 100 mm if no narrow filters, such as U and CH\(_4\), are used. Table I shows the relevant characteristics of the filters used in our observatory. The camera used is a DMK-41AU02\(^6\) for planetary and solar observation, with an array of 1280x960 pixels with 8 bit dynamic range and 15 fps, which is suitable for the observation of bright objects. Our observations are available to the interested reader through the companion website.

3.2. Photometric calibration

Let’s assume we have now data on how bright a particular location (or set of locations) looks in our images. However, in order to model the reflectivity what we need is information about intrinsic properties of the atmosphere studied, i.e. planet absolute reflectivity \(I/F\). So we need to calibrate the image in order to translate the photometric information \(DN\) read from the images to absolute reflectivity \((I/F)_{obs}\) at each point. For simplicity, we will use well known \(I/F\) reference values from the bibliography. In particular, we are here interested in the absolute reflectivity dependence on latitude, since rapid rotation of Jupiter averages zonally (i.e. in the East-West direction) its main properties related to the vertical cloud structure.

In the case of Jupiter, the absolute reflectivity spectrum \(I/F(\lambda)\) is known at different latitudes (Chanover et al., 1996)\(^7\), so comparing the photometry from an image taken with a certain filter, with the known values of \(I/F\) for the filter characteristic wavelength interval, a ‘calibration factor’ can be obtained. This factor can then be used to translate the photometry value \(DN\) of every image point to a value of absolute reflectivity \((I/F)_{obs}\) calibrating the whole image. One possible method to obtain this calibration factor is based on directly comparing the area of the \(DN\) vs. latitude along the central meridian in our planet image with that of \(I/F\) reference values vs. latitude for the correspondent filter wavelength. In this way every point \((long, lat)\) in our planet disk can be calibrated as follows:

\[
(I/F)_{obs(long, lat)} = \frac{A_{IF(\phi)}}{A_{DN(\phi)}}(DN)_{long, lat}
\]  

(15)

This image calibration process can be easily implemented in most general purpose programming tools like Matlab or IDL. We provide Matlab routines for this step in the companion website. Obviously, this method implies the assumption that the overall intrinsic brightness of the planet does not change substantially over time.

3.3. Data fitting and model parameters retrieval

Once the images have been calibrated, we can compare these observed \((I/F)_{obs}\) values with those \((I/F)_{mod}\) predicted by the RLM model given by equation (9), in order to finally get information about the planetary upper atmosphere structure.

\[\text{http://www.planewave.com/index.php?page=1&id0=0&id=2}\]

\[\text{http://www.ehu.es/aula-espazio/presentacion.html}\]


\[\text{http://charon.nmsu.edu/~nchanove/jupcal/jupitercal.html}\]
As explained in section 2, at a given latitude and wavelength the RLM model predicts the absolute reflectivity \((I/F)_{\text{mod}}\) value (if \(R_\lambda\) and pressure \(P\) values, i.e. the optical depth, were known), so a curve \((I/F)_{\text{mod}}\) vs. longitude can be built for that latitude and wavelength.

Comparing at each analyzed latitude these model based predicted \((I/F)_{\text{mod}}\) vs. longitude curves with those observed \((I/F)_{\text{obs}}\) vs. longitude ones from our calibrated images at the corresponding wavelength, we can get an estimation of cloud reflectivity spectrum \(R_\lambda\) (at the wavelength values corresponding to the filters used) and pressure \(P\) allowing the best match between both predicted and observed curves. This is a kind of inverse problem deducing the atmosphere structure (\(R_\lambda\) spectrum and pressure \(P\) at each latitude) from the resulting absolute reflectivity by comparing predicted and observed values. Our approach will be extremely simple: we will scan the whole free parameter (\(R_\lambda\) and \(P\)) space. This is only feasible in cases like this, when the computation time is short enough.

The ‘quality’ of the fit between predicted and observed \((I/F)\) vs. longitude curves can be visually checked by representing the curves for specific values of \(R_\lambda\) and \(P\) parameters (an Excel file is provided in the companion website allowing this manual fitting process). Nevertheless, for a systematic quantitative estimation of the quality of the fit for many different values of these parameters, we can define a ‘misfit function’ \(\chi^2(P,R_\lambda)\) based on quadratic deviation (Pérez-Hoyos et al., 2005):

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{(I/F)_{\text{obs}}(i) - (I/F)_{\text{mod}}(i)}{\sigma_i^2} \right)^2
\]

(16)

where the sum is extended over all the longitude values. In the expression above \(\sigma_i^2\) is a reference value for the error estimation, which has been set at 10% of the maximum \(I/F\) value in the calibrated image at this latitude. This function simply throws values below 1 (i.e., below assumed error) for acceptable models and above 1 for models far from observations. This is calculated for every possible combination of the free parameters \(P\) and \(R_\lambda\) (the latter for each filter). The only thing we have to do is to find the minimum of the total error function \(\Sigma\chi^2\) (considering all our five images) defined over the free parameter space.

Alternatively, in order to get more realistic features in the overall results some additional constraints can be imposed during this atmosphere characterization process. For example, the similarity between the reflectivity \(R\) at red and methane wavelengths, which is a known empirical fact (West et al., 2004; Pérez-Hoyos et al., 2005; Karkoschka & Tomasko, 2005), can be considered by imposing the condition \(R_{R}=R_{CH4}\). This condition can be imposed by considering the sum of the misfit functions for red and methane images.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Date</th>
<th>Time (UT)</th>
<th>Recording time (s)</th>
<th>Number of frames</th>
<th>Stacked frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>2011-10-26</td>
<td>1:06</td>
<td>175</td>
<td>206</td>
<td>180</td>
</tr>
<tr>
<td>B</td>
<td>2011-10-26</td>
<td>1:17</td>
<td>130</td>
<td>808</td>
<td>600</td>
</tr>
<tr>
<td>G</td>
<td>2011-10-26</td>
<td>1:16</td>
<td>130</td>
<td>809</td>
<td>500</td>
</tr>
<tr>
<td>R</td>
<td>2011-10-26</td>
<td>1:02</td>
<td>130</td>
<td>807</td>
<td>500</td>
</tr>
<tr>
<td>CH4</td>
<td>2011-10-26</td>
<td>1:24</td>
<td>60</td>
<td>59</td>
<td>59</td>
</tr>
</tbody>
</table>

Common data for all images: \(B = 3.36^\circ, B' = 3.05^\circ, \alpha = 0.73^\circ\)
Both alternative approaches (with and without this condition) have been used for the case of Jupiter atmosphere, as discussed in the next section, and implemented in Matlab routines also provided in the website.

4. Results for Jupiter’s bands: cloud altitude and spectral reflectivity

Jupiter’s atmosphere is characterized by the presence of different horizontal stripes of stable zonal winds aligned parallel to the planet equator, showing alternative dark belts and brighter zones clearly visible through a telescope (Rogers, 1995). We want to determine this overall atmosphere structure in terms of pressure levels and reflectivity spectrum by using the above described methodology.

This methodology has therefore been applied to Jupiter based on telescope observations from the Aula EspaZio Observatory in Bilbao on October 26\textsuperscript{th} 2011, when Jupiter was at a minimum distance from Earth (3.97 AU) providing a resolution of around 1000 km/pixel. Of course this resolution cannot be actually achieved due to Earth atmospheric turbulence that can only partially be improved by image processing.

The five videos (U, B, G, R, CH\textsubscript{4}) were processed in REGISTAX leading to the images in figure 4, yet for visualization purposes some additional processing was done to better show the atmosphere details. In the context of RLM model brighter bands can be interpreted as corresponding to clouds with higher reflectivity and/or located at lower pressure levels (sunlight therefore undergoing less absorption/scattering processes).

After calibrating the five images the RLM parameters ($P$, $R_\lambda$) are estimated as described in previous section. The RLM model is so computationally inexpensive that the whole free parameter space can be modeled without substantial effort. For example, error functions at 20°N latitude are represented in figure 5, showing their dependence with pressure $P$ and reflectivity $R_\lambda$ parameters. Additionally, the error function $\chi^2_{\text{RCH}_4}$, imposing the condition $R_R=R_{\text{CH}_4}$, is also shown for the alternative approach problem resolution. The global minimum of the error function has to be found providing the combination of parameters which best fits observations at every latitude.

![Figure 4](image.png)

Figure 4. Jupiter images taken with U, B, G, R, CH\textsubscript{4} filters and processed with REGISTAX
In the original approach, without imposing any additional condition, we use the error functions \( \chi^2_{U}, \chi^2_{B}, \chi^2_{G}, \chi^2_{R}, \chi^2_{CH_4} \) to estimate the Jupiter atmosphere characterization at every degree of latitude. Figure 6 represents both the RLM model based predicted \((I/F)_{\text{mod}}\) vs. longitude curves (lines) and those observed \((I/F)_{\text{obs}}\) vs. longitude ones from the five calibrated images (dots) at 20°N latitude. It is remarkable that both model predicted and observed \(I/F\) values fit particularly well, considering the simplicity of the RLM model used.

In the same way, pressure and reflectivity spectrum can be estimated for all latitudes. Figure 7 represents the final result of this analysis; showing for all the latitudes analyzed the minimum error values of pressure \(P\), the reflectivity spectrum \(R\), and finally the total error \(\Sigma \chi^2\). Results are presented for latitudes from -60° to 60° where the images provide enough information for a proper analysis. Error bars represent the limit values where the total error \(\Sigma \chi^2\) (in the case of pressure \(P\)) or the respective error \(\chi^2\) (in the case of reflectivity \(R\)) is 50% higher than the respective minimum values.

Averaged reflectivity values over all latitudes leads to a reflectivity spectrum shown in figure 8 (left). If the condition of equality of reflectivity at red and methane wavelengths is imposed as ‘empirical condition’ we get the reflectivity spectrum in the right panel. Imposing this condition avoids the discrepancy of reflectivity in red and methane wavelengths, but forces a non-optimal \(R\) value for methane, leading to higher pressure values and slightly higher errors \(\chi^2\) when fitting \(I/F\) curves.

If figure 7 is compared with figure 4 the main bands in Jupiter atmosphere can be recognized from this photometric analysis. In addition, for the pressure values profile of ammonia high clouds top layer in Jupiter, values of around 100 to 200 mbar shown in figure 7 (top) as predicted by the proposed methodology, are acceptable according to the scientific literature (e.g., West et al. 2004).

---

**Figure 5.** Error functions \( \chi^2_{U}, \chi^2_{B}, \chi^2_{G}, \chi^2_{R}, \chi^2_{CH_4} \) showing their dependence on \(P\) and \(R\) in images \(U,B,G,R,CH_4\) and \(\chi^2_{RCH_4}\) imposing ‘\(R_B = R_{CH_4}\)’ condition. Latitude 20°N
Figure 6. Model predicted (solid line) and observed (grey dots) $I/F$ vs. Longitude, at latitude 20°N. Pressure 150mbar; Reflectivity spectrum: $R_U=0.67$, $R_B=0.62$, $R_G=0.69$, $R_R=0.72$, $R_{CH_4}=0.25$

Figure 7. Jupiter atmosphere structure: $P$, $R_A$ spectrum and Error vs. Latitude

5. Conclusion
A methodology for the photometric based characterization of planetary upper clouds has been defined and validated for the case of Jupiter, leading to remarkable results given the simplicity of the radiative model used.

A sensitivity analysis of the results depending on the main aspects determining the quality of the images, such as the seeing or number of frames, could be performed, as well as for different calibration methods. In addition, taking into consideration that the radiative transfer model RLM used only takes into account Rayleigh scattering and absorption phenomena, more elaborated models could be analyzed, such as some existing empirical models. However, even with all these limitations the methodology proposed provides a very acceptable characterization of the upper layers of Jupiter atmosphere, and the activity here described can easily be performed with affordable equipment from an urban environment.
On the other hand, this methodology was also applied to the case of Saturn with reasonable results, but the applicability of this approach to other giant planets like Uranus and Neptune would require larger telescope. In addition, this methodology can also be applied to the case of Venus, where the CO$_2$ absorption phenomena can be reproduced.

Acknowledgements
This work was supported by the Spanish MICIIN Project AYA2009-10701 with FEDER and Grupos Gobierno Vasco IT-464-07. The Aula EspaZio is supported by the Diputacion Foral de Bizkaia/Bizkaiko Foru Aldundia, the Escuela Técnica Superior de Ingeniería de Bilbao and the Universidad del País Vasco.

Appendix. Supplementary on-line material
The necessary material to perform the activity described above is available for download at http://www.ajax.ehu.es/photometry/ including:

- **Jupiter Images**:
  - videos (AVI) in U,B,G,R,CH$_4$ filters,
  - the correspondent five REGISTAX processed images (FITS),
  - photometric scans files at different latitudes and along central meridian for the five images (ASCII files),
  - the five calibrated images (Matlab .mat files).

- **Matlab functions**:
  - filters characterization,
  - images calibration,
  - RLM model adjustment for atmosphere characterization,
  - cross-section, optical depth and local gravity calculation.

- **Additional information and tools**:
  - methane spectrum by E. Karkoschka,
  - sensitivity curves for DMK-41AU02 camera and filters used,
  - Jupiter’s reflectivity (I/F) curves for the filters used from Chanover data,
  - Excel tool for playing with RLM model,
  - CORTES tool developed by JF Rojas, complementary to WinJupos for photometry reading.

---

**Figure 8.** Averaged reflectivity $R$ spectrum and standard deviations without (up) and with (down) $R=R_{CH_4}$ condition

---

8 Alternative sources of planetary images are also available: PVOL, IOPW, ALPO (http://www.ehu.es/iopw/)
References


(London, UK).